# Quantum mechanics and quantum computing

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# Quantum mechanics

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# Quantum mechanics

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### 1.2.1 Mixed quantum states

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### 1.2.3 Probability

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### 1.3.1 Indexing states to time

We have state defined at each time $t$.

$\Psi(t)$.

### 1.3.2 Wave functions

We have state $\Psi(t)$.

$\psi(x, t) = \langle x|\Psi(t)\rangle$

This is the wave function.

### 1.3.3 Schrodinger

#### Discrete time

With discrete time we can use a canonical operator for moving between discrete states in single jumps.

With discrete time there must a countable number of states.

We can index time to the integers.

At time 0 we have $v$

At time 1 we have $\Psi v$

At time 2 we have $\Psi\Psi v$

We can write this as $\Psi(t_1, t_0) = \Psi^{t_1 - t_0}$

### 1.3.4 Representation theory for the time group

Time is a linear operator

Instead, we describe the time operator as a Lie group, using Lie algebra.

$\Psi(t_b - t_a) = e^{(t_b - t_a)X}$

#### States are vectors

We can remove a degree of freedom by using norm of 1 for vectors

For each dynamic system we define a set of possible states.

We can describe a state $v \in V$.

#### Finite state spaces

We can describe a system like heads or tails.

#### Infinite state spaces

This can describe continous position, or an angle.

### 1.3.5 Indexing time to the real numbers

Sloan's theorem

### 1.3.6 Continous time with Lie algebra

We use $X = iH$, what are the implications of this compared to other choices?

Lie algebras with $n \times x$

This loops back? multiple dimensions, infinite, so maybe not?

With continuous time we do not have a single operator to describe movements. There is always one smaller.

With continous time there must be either a single state, or an uncountably infinite number of states.

$U = M_n^n$

$U = (I + \frac{1}{n}G_n)^n$

$U = \lim_{n\to\infty}(I + \frac{1}{n}G)^n$

Now:

$UU^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G)^* = I$

$(I + \frac{1}{n}G)(I + \frac{1}{n}G^*) = I$

$G = -G^*$

$G = iH$

$iH = -(iH)^*$

$H = H^*$

$H$ is Hermitian

$U = \lim_{n\to\infty}(I + \frac{1}{n}iH)^n$

This isn't quite right, need defined for different time jumps.

### 1.3.7 Unitary time

Why? What's the interpretation here? Is this an assumption, or just a modelling choice?

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)X^*}e^{(t_b-t_a)X}$

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)(X^*+X)}$

$X = iH$

$\Psi(t_b - t_a)^*\Psi(t_b - t_a) = e^{(t_b-t_a)(-iH+iH)} = I$

$\Psi(t_b - t_a) = e^{(t_b - t_a)iH}$

### 1.3.8 The time-depedendent general Schrödinger equation

$v(t_b) = e^{(t_b - t_a)X} v(t_a)$

$v(t+\delta) = e^{\delta X} v(t)$

$v(t+\delta) = (I + \delta X)v(t)$

$\frac{v(t+\delta) - v(t)}{\delta} = Xv(t)$

$\frac{\delta v(t)}{\delta t} = Xv(t)$

$\frac{\delta v(t)}{\delta t} = iHv(t)$

### 1.3.9 The energy operator and the time-indepedendent general Schrödinger equation

$E = ih\frac{\delta}{\delta t}$

$Ev(t) = Hv(t)$

## 1.4 Infinite dimensional quantum states

### 1.4.1 Position

### 1.4.2 Velocity

### 1.4.3 Momentum

### 1.4.4 Moving to 3 dimensions

### 1.4.5 The action integral

### 1.4.6 Renormalisation

## 1.5 Quantum entanglement

## 1.6 Other

### 1.6.1 The Hamiltonian of quantum mechanics

### 1.6.2 Plank's constant

We can add Plank's constant, due to the arbitrary scaling of time.

### 1.6.3 Phase shift

### 1.6.4 Density matrix

### 1.6.5 Born rule

### 1.6.6 Spin-statistics theorem

### 1.6.7 Heisenberg's uncertainty principle

Result of spin-statistics theorem?

### 1.6.8 The Dirac equation

### 1.6.9 The quantum harmonic oscillator

# Chapter 2

# Quantum Field Theory (QFT)

## 2.1

# Chapter 3

# The hydrogen atom

## 3.1 Introduction

### 3.1.1 Atomic states

transition matrix can be v complex

### 3.1.2 Factored states

+ can store rules for simply

finite atomic: permutation matrix inf atomic: how to represent?

basis state dynamic to comp sci??

factored: how to model atomic as factored?