General equilibrium

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Preface

This is a live document, and is full of gaps, mistakes, typos etc.

Part I

Exchange economies

Chapter 1

Edgeworth boxes

1.1 Trade

1.1.1 Edgeworth box

So now we have a framework for how agents can interact. We want to model trade, what games can do this?

To do this we introduce the Edgeworth box. This is a rectangle where the length of the x and y axes represent the total amount of those goods, and points on the box represent allocations of the goods. There is an initial endowment of goods.

$x = \{0.2, 0.8\}$

$y = \{0.8, 0.2\}$

[Put box here]

Their respective utility functions are:

$u_a = f_a(x_a, y_a)$

$u_b = f_b(x_b, y_b)$

In this example we use:

$u_{a,b} = x^{0.5}y^{0.5}$

We can add indifference curves to this box, which intercept at the endowment.

[Put box here]

The agents would be better off if they could trade so that they both had half of a unit of each good.

$x = \{0.5, 0.5\}$

$y = \{0.5, 0.5\}$

However they could also both be better off with

$x = \{0.6, 0.6\}$

$y = \{0.4, 0.4\}$

[Box here]

There are many such trades which could be made, which agents would rank differently.

There are also points where no further trade would be agreed by both parties. For example with the above outcome, any further trade would make at least one party worse off.

Such points are called Pareto efficient. That is, if a point is not Pareto efficient at least one party can be made better off without harming others.

[Box here]

Chapter 2

Commodity markets

2.1 Exchange design

2.1.1 Tick size

2.1.2 Latency

2.1.3 Pre-trade transparency

2.1.4 Post-trade transparency

2.1.5 Circuit breakers

2.1.6 Public exchanges

2.1.7 Dark pools

2.1.8 Over-The-Counter (OTC)

2.2 Order types

2.2.1 Order book

2.2.2 Passive orders

2.2.3 Stop-loss orders

2.2.4 Resting orders

2.2.5 Limit orders

2.2.6 Wash trading

2.2.7 Orders on multiple exchanges

2.3 Intermediaries

2.3.1 Brokers

2.3.2 Front running

2.3.3 Payment for order flow

2.4 Measures

Chapter 3

Consumer choice in one period

3.1 Utility functions recap

3.1.1 Utility functions

We have $U = f(\mathbf{x})$.

Throughout this we will be using a Cobb-Douglas utility function, and discuss the properties of other utility functions at the end.

We have $U = \prod_i x_i^{\alpha_i}$.

3.1.2 Marginal utility

The marginal utility of product x_1 is:

$\frac{\delta}{\delta x_1} f(\mathbf{x})$

For Cobb-Douglas:

$U = \sum_i x_i^{\alpha_i})$.

$\frac{\delta}{\delta x_1} f(\mathbf{x}) = \frac{\delta}{\delta x_1} \prod_i x_i^{\alpha_i}$

$\frac{\delta}{\delta x_1} f(\mathbf{x}) = \frac{1}{x_1} \alpha_1 \prod_i x_i^{\alpha_i}$

3.1.3 Indifference curves

We have $U = f(x, y)$

An indifference curve is a curve where a consumer is indifferent to all points on it.

$f(x,y) = c$

3.1.4 Marginal rate of substitution

The marginal rate of substitution is the amount of one good that a customer is willing to give up for another.

This is the gradient of the indifference curve.

$MRS(x_1, x_2) = \frac{MU(x_1)}{MU(x_2)}$

$MU(x_1) = \frac{1}{x_1}\alpha_1 \prod_i x_i^{\alpha_i}$ $MU(x_2) = \frac{1}{x_2}\alpha_2 \prod_i x_i^{\alpha_i}$

$MRS(x_1, x_2) = \frac{\frac{1}{x_1}\alpha_1 \prod_i x_i^{\alpha_i}}{\frac{1}{x_2}\alpha_2 \prod_i x_i^{\alpha_i}}$

$MRS(x_1, x_2) = \frac{\frac{1}{x_1}\alpha_1}{\frac{1}{x_2}\alpha_2}$

$MRS(x_1, x_2) = \frac{\frac{\alpha_1}{x_1}}{\frac{\alpha_2}{x_2}}$

3.1.5 Elasticity of substitution

3.2 The utility maximisation problem

3.2.1 First-order conditions

We have a utility function:

$U = f(\mathbf{x})$

And the constraint:

$\sum_i (x_i - c_i)p_i \le 0$

This gives us the constrained optimisation problem:

$L = f(\mathbf{x}) - \lambda \sum_i (x_i - c_i)p_i$

The first-order conditions are:

$L_{x_i} = \frac{\delta}{\delta x_i} f(\mathbf{x}) - \lambda p_i = 0$

Or:

$\lambda = \frac{MU(x_i)}{p_i}$

This means for any pair we have:

$\frac{MU(x_i)}{p_i} = \frac{MU(x_j)}{p_j}$

For the Cobb-Douglas utility function the first-order conditions are:

$\frac{\delta}{\delta x_1} f(\mathbf{x}) = \frac{1}{x_1} \alpha_1 \prod_i x_i^{\alpha_i}$

$L_{x_i} = \frac{\delta}{\delta x_i} f(\mathbf{x}) - \lambda p_i = 0$

3.2.2 Marshallian demand

We can write a demand function:

$x_i = f(I, \mathbf{p})$

We can derive this from the first-order conditions of a specific utility function.

3.2.3 Own-price elasticity of demand

We have our Marshallian demand function:

$x_i = x_{di}(I, \mathbf{p})$

The derivative of this with respect to price is the additional amount consumed after prices increase.

$\frac{\delta}{p_i} x_{di}(I, \mathbf{p})$

For the Cobb-Douglas utility function, this is:

$\frac{\delta}{p_i} x_{di}(I, \mathbf{p})$

In addition to the derivative, we may be interested in the elasticity. That is, the proportional change in output after a change in price.

$\xi_i = \frac{\frac{\Delta x_i}{x_i}}{\frac{\Delta p_i}{p_i}}$

$\xi_i = \frac{\Delta x_i}{\Delta p_i} \frac{p_i}{x_i}$

For the point-price elasticity of demand we evaluate infintesimal movements.

$$\xi_i = \frac{\delta x_i}{\delta p_i} \frac{p_i}{x_i}$$

3.2.4 Constant price elasticity of demand

If the point-price elasticity of demand is constant we have:

$$\xi_i = \frac{\delta x_i}{\delta p_i} \frac{p_i}{x_i} = c$$

This means that small changes in the price at low level cause large changes in quantity.

3.2.5 Arc-price elasticity of demand

We may have price changes which are non-infintesimal.

$$E_d = \frac{\Delta Q/\bar{Q}}{\Delta P/\bar{P}}$$

Where $\bar{Q}$ and $\bar{P}$ are the mid-points between the start and end.

3.2.6 Super elasticty of demand

If elasticity is constant super elasticity is 0.

3.2.7 Veblen goods

The demand curve is sloping up.

3.3 Cross-price elasticity of demand, complements and substitutes

3.3.1 Cross-price elasticity of demand

We have our Marshallian demand function:

$$x_i = x_{di}(I, \mathbf{p})$$

The derivative of this with respect to price is the additional amount consumed after prices increase.

$$\frac{\delta}{p_i} x_{di}(I, \mathbf{p})$$

3.3.2 Complements

3.3.3 Substitutes

3.3.4 Diversion ratio

3.4 Income effects

3.4.1 Engel curves and income elasticity of demand

The Engel curve shows demand for a good as a function of income.

Derivative $x_{di} = x_{di}(I, \mathbf{p})$

$\frac{\delta}{\delta I} x_{di}(I, \mathbf{p})$

Income elasticity of demand $\xi_i = \frac{\delta x_{di}}{\delta I} \frac{I}{x_{di}}$

3.4.2 Normal goods

As income rises, demand also rises.

This is the same as saying the income elasticity of demand is above 0.

$\xi_i = \frac{\delta x_{di}}{\delta I} \frac{I}{x_{di}}$

3.4.3 Inferior goods

An inferior good is one where the demand falls as income increases.

This is the same as the income elasticity of demand being less than 0.

3.4.4 Necessities

Necessities are goods which increase in demand as income rises, but by a smaller proportion.

The income elasticity of demand is between 0 and 1.

3.4.5 Luxuries

Luxuries are goods which increase in demand as income rises, by a larger proportion.

The income elasticity of demand is above 1.

3.4.6 Ordinary goods

As price rises, demand goes down.

3.4.7 Giffen goods

As price rises, demand goes up. Not because of the slope of the demand curve, but because the income effect and inferior effect are strong.

Substitution means you buy less.

Income means you buy less generally, but move towards inferior goods.

3.4.8 Income effect

3.4.9 Slutsky equation

3.5 Indirect utility functions

3.5.1 Indirect utility functions

The normal utility function is:

$U = f(\mathbf{x})$

We have our demand:

$x_{di} = x_{di}(I, \mathbf{p})$

3.5.2 The indirect utility function

We can plut this in to get:

$U = g(I, \mathbf{p})$

3.6 The expenditure minimisation problem

3.6.1 The expenditure function

3.6.2 Shephard's lemma

3.6.3 Hick's demand

3.6.4 Roy's identity

3.7 Specific utility functions

3.7.1 Cobb-Douglas utility function

$U = A\sum_i X_i^{\alpha_i}$

3.7.2 Leontief utility function

$U = \sum_i X_i^{\alpha_i}$

3.7.3 Constant Elasticity of Substitution (CES) utility function

For some constant r.

$U = A[\alpha_i X_i^r]^{\frac{1}{r}}$

3.7.4 Almost Ideal Demand System

3.7.5 Representative consumer

3.8 Other

3.8.1 Consumer surplus

3.8.2 Aggregating to demand curves

Aggregating individual preferences to the demand curve for a product

Move Representative consumers here.

Chapter 4

Consumer choice under uncertainty

4.1 Uncertainty

4.1.1 Expected utility theory

Part II

Intertemporal consumer theory

Chapter 5

Saving, stockpiling and risk-free debt

5.1 Saving

5.1.1 Savings

Alice starts cutting back on meat to save shells for the future. This gives Bob a dilemma; he has less income so he can either:

- Continue spending, drawing down on his shells; or
- Spend less (for simplicity, on fish).

In the first case Alice's savings are equal to Bob's "borrowing", and net saving is zero. In the second case, Alice can't get the extra shells to save, because Bob is not buying her fish. Net savings are also zero.

This is cash saving. In the real world total savings can be above zero because of stockpiling and investment.

5.2 Stockpiling

5.2.1 Stockpiling

Let's consider what happens if Alice tries to save money by producing more fish. If she uses the additional shells from this to buy more goods, she hasn't saved. If she doesn't sell them she hasn't acquired any additional shells.

While building up stockpiles isn't giving Alice more money now, the economy is producing more that it is consuming, and so this is an investment in the future.

5.3 Saving

5.3.1 Securities

Alice wants to build a new fishing rod to catch more fish, but doing so will require a lot of resources from others. She has to persuade others to lend her shells, and in return she promises to pay back more shells in future with all extra shells she'll get from selling more fish.

The other villagers are now saving. They are consuming less, and this surplus is being used by Alice. Note that the villagers are not necessarily holding a different amount of shells as a result of this activity. The value, in shells, of this investment is equal to the value of savings.

5.4 Interest rates

5.4.1 Securities

Annual returns/yield/interest rates

Bonds and coupons

Maturity

Internal rate of return

Discounted cash flow

debt structuring, refinancing after rates change, terms balanaced over time v focused on one period?

5.5 Debt in different currencies

5.5.1 Covered interest arbitrage

5.5.2 Uncovered interest arbitrage

Chapter 6

Intertemporal consumer choice

6.1 Discounted utility

6.1.1 Discounting

Time inconsistency (hyperbolic discounting). again, this should mirror ai. page overview if necessary. call it that?

6.2 Intertemporal budget constraint

6.2.1 Budget constraint

$\sum_{t=T} C^t(1+r)^{-t} = \sum_{t=T} Y_t(1+r_t)^{-t} + W_T$

W_t is wealth endowment at time T.

6.2.2 The Euler equation

If we have exponential discounting we have:

$U_T = E[\sum_{t=T}^{\infty}(1+\delta)^t U(C_t)]$

The first-order conditions give us:

$u'(x_t) = (1+\delta)(1+r_t)u'(x_{t+1})$

6.2.3 The Euler equation with liquidity constraints

$u'(x_t) = (1+\delta)(1+r_t)u'(x_{t+1}) + \lambda_{t+1}$

6.2.4 The Euler equation with continuous time

6.2.5 Marginal propensity to consume

6.3 Renewal

6.3.1 Switching costs

6.4 Household finance

6.4.1 Household wealth and liquidity

Hold cash, equity, bonds, mortgages.

6.4.2 Mortgages

6.4.3 Fixed and variable rates

6.4.4 Expense and income timing

6.5 Other

6.5.1 Elasticity of intertemporal substitution

6.5.2 Habit formation

Different utility function.

6.5.3 Durable goods

Can wait to purchase. depends on expected prices in future.

6.5.4 Buying or owning

production of other commodities. house produces rentable space eg if you own house, you have rentable space each period.

depreciation? 1 for something like rent, maybe 0.01 for long term asset

Chapter 7

Insurance

7.1 Insurance

7.1.1 Diviserifiable and systemic risk

7.2 Pensions

7.2.1 Workplace and non-workplace pensions

Chapter 8

Risky debt

8.1 Pricing

8.1.1 Securities pricing

8.1.2 Hedging

8.1.3 Risk-free rates

8.1.4 Capital Asset Pricing Model (CAPM)

8.1.5 Arbitrage Pricing Theory (APT)

8.1.6 Martingale pricing

8.1.7 Put-call parity

8.1.8 Yield curve

8.1.9 Secured and unsecured debt

8.1.10 Instruments

Debt types: inflation protected, interest linked (SVR etc)

Chapter 9

Derivative markets

9.1 Introduction

9.1.1 Money markets

9.1.2 Counterparties

Counterparties and counterparty risk.

9.1.3 Derivatives

9.1.4 Naked short selling

9.1.5 Options

9.1.6 Calls and puts

9.1.7 Strike price

9.1.8 Short selling

9.1.9 European options

9.1.10 American options

9.1.11 Options pricing

9.1.12 Black-Scholes equation

9.1.13 Black-Scholes formula

Part III

Producer theory

Chapter 10

Production and intermediate goods

10.1 Production functions

10.1.1 Production functions and marginal products

Production functions

A firm produces Q using inputs X.

$Q = f(X_1, ..., X_n)$

Marginal products

This is the marginal utility, adapted for the production setting.

$MP = \frac{\delta}{\delta x_1} f(\mathbf{x})$

Diminishing marginal returns

This says that marginal returns decrease as the use of a factor increases.

$\frac{\delta^2}{\delta {x_1}^2} f(\mathbf{x}) < 0$

10.1.2 Marginal and average costs

10.1.3 Average total cost

10.1.4 Long-run average incremental cost

10.1.5 Isoquants

Isoquants are indifference curves for firms.

We have a production function: $Q = f(X)$.

An isoquant is defined for each c $f(X) = c$, where X is a vector.

10.1.6 Marginal rate of technical substitution

This is the marginal rate of substitution, adapted for firms.

$MRTS =$

10.1.7 Marginal and average costs

10.1.8 Average total cost

10.1.9 Long-run average incremental cost

10.2 Choosing production inputs

10.2.1 Isoquants

Isoquants are indifference curves for firms.

We have a production function: $Q = f(X)$.

An isoquant is defined for each c $f(X) = c$, where X is a vector.

10.2.2 Marginal rate of technical substitution

This is the marginal rate of substitution, adapted for firms.

$MRTS =$

10.3 Specific production functions

10.3.1 Cobb-Douglas production function

$Q = A \sum_i X_i^{\alpha_i}$

10.3.2 Leontief production function

$Q = \sum_i X_i^{\alpha_i}$

10.3.3 Constant Elasticity of Substitution (CES) production function

For some constant r.

$Q = A[\alpha_i X_i^r]^{\frac{1}{r}}$

10.4 Input-output tables

10.4.1 Input-output tables

10.5 Specific production functions

10.5.1 Marginal rate of transformation

10.5.2 Production-Possibility Frontier

10.5.3 Other

Production functions

A firm produces Q using inputs X.

$Q = f(X_1, ..., X_n)$

Marginal products

This is the marginal utility, adapted for the production setting.

$MP = \frac{\delta}{\delta x_1} f(\mathbf{x})$

Diminishing marginal returns

This says that marginal returns decrease as the use of a factor increases.

$\frac{\delta^2}{\delta {x_1}^2} f(\mathbf{x}) < 0$

10.5.4 Passthrough

10.5.5 Monopsony and monopoly power

Chapter 11

The Robinson Crusoe model

11.1 Introduction

Chapter 12

Basic general equilibrium

12.1 Introduction

12.2 Productive efficiency

12.2.1 Production-possibility frontier

12.2.2 Productive efficiency

12.3 Allocative efficiency

12.3.1 Allocative efficiency

12.3.2 Pareto efficiency

12.3.3 Kaldor-Hicks efficiency

Part IV

Welfare economics

Chapter 13

Taxation

13.1 Introduction

13.1.1 Diamond–Mirrlees

Chapter 14

Externalities

14.1 Introduction

Chapter 15

Public goods

15.1 Introduction

15.1.1 Ramsey pricing

Chapter 16

Welfare economics

16.1 Introduction